Unified Engineering Fall 2007
Fluids Problem F7

Air flows into a channel of height h, at uniform speed V_1, as a thin jet of height $h/8$. The air then mixes in the channel into a uniform velocity V_2 at the outlet. The density ρ is constant everywhere. There is a uniform pressure p_1 over the inlet plane, and a uniform pressure p_2 over the outlet plane.

a) Apply the mass conservation integral to the dashed control volume shown in the figure,

$$\oint \rho\,(\vec{V}\cdot\hat{n})\,dA \;=\; 0$$

and thus determine the exit velocity V_2 in terms of the known inlet maximum velocity V_1.

b) Apply the momentum conservation integral to the dashed control volume,

$$\oint \left[\rho\,(\vec{V}\cdot\hat{n})\,\vec{V} \;+\; p\hat{n}\right]\,dA \;=\; 0$$

and thus determine the pressure difference $p_2 - p_1$ in terms of ρ and V_1. State whether the mixing causes the pressure to increase or decrease downstream.